B.Tech. DEGREE EXAMINATION, NOVEMBER 2017

Third/ Fourth/ Fifth Semester

15MA302 – DISCRETE MATHEMATICS

(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:

(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.

(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

1. $p \to q$ is logically equivalent to
   (A) $\neg p \to \neg q$
   (B) $\neg q \to \neg p$
   (C) $p \to \neg q$
   (D) $\neg p \to q$

2. $p \to (q \to p) \equiv$
   (A) T
   (B) F
   (C) $p \to q$
   (D) $p \wedge q$

3. The dual of $(\neg p \vee q) \wedge F$ is
   (A) $(p \vee q) \wedge T$
   (B) $(\neg p \wedge q) \vee F$
   (C) $(p \to q) \wedge F$
   (D) $(\neg p \to q) \vee T$

4. The negation of "every city in Tamilnadu is clean".
   (A) Every city in Tamilnadu is not clean
   (B) Some city in Tamilnadu are not clean
   (C) Every city in Tamilnadu is clean
   (D) Every city in Tamilnadu are dirty

5. A relation R from a non-empty set A to a non - empty set B is a
   (A) Subset of A×B
   (B) Subset of A×A
   (C) Subset of B×B
   (D) Subset of B×A

6. In a poset, the greatest and least element, if they exist, are
   (A) Exactly two
   (B) Zero
   (C) Unique
   (D) More than one

7. If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ and $f = \{(1, a), (2, b), (3, c), (4, b)\}$, then the function is
   (A) Neither 1 – 1 nor onto
   (B) 1 – 1 but not onto
   (C) Onto but not 1 – 1
   (D) Both 1 – 1 and onto

8. Assuming that repetitions are not permitted, how many four-digit numbers can be formed from 1, 2, 3, 4, 5, 6…..
   (A) 360
   (B) 15
   (C) 20
   (D) 460

9. The generating function of the sequence 1, 1, 1..... is given by

(A) $(1+x)^{-1}$
(B) $(1+x)^{-2}$
(C) $(1-x)^{-1}$
(D) $(1-x)^{-2}$

10. The recurrence relation of Fibonacci sequence is

(A) $F_n = F_{n-1} + F_{n-2}, n \geq 0$
(B) $F_n = F_{n-1} - F_{n-2}, n \geq 0$
(C) $F_n = F_{n-1} + F_{n-2}, n \geq 2$
(D) $F_n = F_{n-1} - F_{n-2}, n \geq 2$

11. The generator of a cyclic group $\{1, -1, i, -i\}$ is

(A) $1, i$
(B) $-1, -i$
(C) $i, -i$
(D) $1, -i$

12. The inverse element of any element "a" in the group of integers Z with the operator * defined by $a * b = a + b + 2 \forall a, b \in Z$ is

(A) −2
(B) 2
(C) a+4
(D) −a −4

13. A vertex with zero indegree is called as

(A) Sink
(B) Source
(C) Terminal
(D) Out degree

14. The value of the prefix expression $+ - \uparrow 32 \uparrow 23 / 8 - 42$ is

(A) 0
(B) 5
(C) −5
(D) 2

15. A connected graph without any circuit is called as

(A) Leaf
(B) Flower
(C) Tree
(D) Loop

16. A maximum height of a 11 vertex binary tree is

(A) 4
(B) 5
(C) 3
(D) 6

17. All Boolean algebras of order 2^n are

(A) Isomorphic to each other
(B) Homorphic to each other
(C) Non-isomorphic to each other
(D) Non-homomorphic to each other

18. Dominance laws are

(A) $a+1=0$ and $a.0=1$
(B) $a+1=1$ and $a.0=0$
(C) $a+1=a$ and $a.0=a$
(D) $a+a=2a$ and $a.0=0$

19. In a lattice $\{L, \leq\}$ $a \vee b = b$

(A) if and only if $a \leq b$
(B) if and only if $a = b$
(C) if and only if $a \geq b$
(D) if and only if $b \leq b$

20. Every finite lattice is

(A) Bounded
(B) Unbounded
(C) Infinite lattice
(D) Uncountable lattice

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Construct the truth table for
    $(7p \to 7q) \leftrightarrow (q \leftrightarrow r)$

22. Show that 'r' can be derived from the premises $p \vee q, p \to r, q \to r$.

23. Prove that $(A-C) \cap (C-B) = \phi$ analytically.

24. If R is the relation of $A = \{1, 2, 3\}$ such that $(a, b) \in R$ if and only if $a+b = even$, find the relational matrix $M_R, M_{R^{-1}}$ and M_{R^2}.

25. Show that $\{1, 3, 5, 7\}$ is an abelian group under multiplication modulo 8.

26. If any disconnected graph has exactly two vertices of odd degree, show that there is a path joining these two vertices.

27. In a Boolean algebra, show that $(a+b)' = a'.b'$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. Show that the premises "one student in this class knows how to write programs in JAVA" and "everyone who known how to write programs in JAVA can get a high-paying job" imply the conclusion "someone in this class can get a high-paying job".

(OR)

b.i. Using CP-rule, derive $p \to (q \to s)$ form the premises $p \to (q \to r)$ and $q \to (r \to s)$. (8 Marks)

ii. Using mathematical induction, show that $\angle n \geq 2^{n-1}$ for $n \geq 1$. (4 Marks)

29.a.i Let $R = \{(1,1), (1,3), (1,5), (2,3), (2,4), (3,3), (3,5), (4,2), (4,4), (5,4)\}$ be a relation on a set $A = \{1, 2, 3, 4, 5\}$. Find transitive closure using Warshall's algorithm. (8 Marks)

ii. Draw the Hasse diagram for the partial ordering relation $\{(A, B) / A \subseteq B\}$ on a power set $P(S)$ where $S = \{a, b, c\}$. (4 Marks)

(OR)

b.i Show that composition of invertible functions is invertible.

ii. If we select 10 points in the interior of an equilateral triangle of side 1, show that there must be atleast two points whose distance apart is less than $\frac{1}{3}$.

30. a. Solve the recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = (n+1)2^n$ with $a_0 = 0$ and $a_1 = 1$.

(OR)

b.i Solve $a_n - 3a_{n-1} = 1, n \geq 1$ and $a_0 = 1$ using generating function.

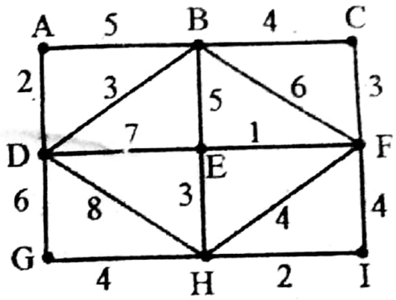
ii. State and prove Lagrange's theorem.

31.a.i Prove that the number of edges in a bipartite graph with n vertices is atmost $\left(\frac{n}{2}\right)^2$.

ii. Construct the binary tree whose in order and post order traversals are DCEBFAHGI and DECFBHIGA respectively.

(OR)

b. Find the minimum spanning tree for the following weighted graph using Kruskal's algorithm.



32.a.i State and prove distributive inequalities in a lattice.

ii. If S_n is the set of all divisors of the positive integer and D is the relation defined by aDb if and only if a divides b, prove that $D_{42} = \{S_{42}, D\}$ is a complemented lattice by finding the complements of all the elements.

(OR)

b.i State and prove DeMorgan's law in Boolean algebra.

ii. **Simplify the Boolean expression** $f(x,y,z) = x\left[y + z\left(\overline{xy + xz}\right)\right]$.

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